within 3% of that computed for uniaxial strain conditions.

III. CURRENT PULSES FROM IMPACT-LOADED PIEZOELECTRIC DISKS

In experiments with the impact-loading technique, planar impact loading is applied to large diameter-to-thickness ratio sample disks whose electrodes are arranged in a guard-ring configuration. The impactor is sufficiently thick that stress is maintained at the impact plane for the duration of the experiment. In regions of the disks in which the electric field and motion are uniaxial, the electrical configuration in the quasistatic uncoupled approximation is as depicted in Fig. 1. With the input stress electrode chosen as the reference potential, the current into a low impedance resistive load which acts as an effective short circuit is, in the uncoupled approximation,[14]

$$i=\frac{\alpha(1-u/U)P_i^\eta A/t_0}{[(1-u/U)t/t_0+\alpha(1-t/t_0)]^2}, \quad 0<t<t_0. \tag{3}$$

In Eq. (3) i is the current in the external circuit connecting the electrodes, α is the ratio of the strained to unstrained permittivity, u is the particle velocity in the region behind the shock front, U is the shock velocity, P_i^η is the piezoelectric polarization, A is the area of the change-collecting electrodes, and t_0 is the transit time of the shock front through the thickness of the disk. The time t is taken to be zero upon first application of the load.

In the present model, the electromechanical coupling effects are neglected. Furthermore, it is assumed that $\epsilon_{ijk}^\eta E_k$ is negligibly small compared to ϵ_{ij}^η and its contribution is ignored. (This assumption has been verified for X-cut quartz.[13]) The electrostrictive effect, manifest as $f_{ijkl}E_j\eta_{kl}$ in Eq. (1b), is treated as a strain-induced change in permittivity and is incorporated into the interpretation of the data through the constant α.

At the time $t=0^+$, the current, called the initial current i_i, is directly related to the piezoelectric polarization by the solution of Eq. (3) for $t=0$,

$$P_i^\eta=\alpha t_0[A(1-u/U)]^{-1}i_i, \quad t=0. \tag{4}$$

Thus, measurement of values for i_i and the shock velocity at known input particle velocities provides a direct measure of the piezoelectric polarization if a value can be determined for α. In the uncoupled approximation α may be determined from measurements of i_i and the current at $t=t_0$, i_f. It follows from Eq. (3) that

$$\alpha=[i_f/i_i]^2[1-u/U]^{-1}. \tag{5}$$

Equations (3)–(5) represent solutions for the current in the uncoupled approximation. Before these equations are used to interpret the experimental results the effect of electromechanical coupling must be considered. Solutions for the current in a fully coupled configuration have been given by Lysne[23] and Thurston.[26] A fully coupled solution suitable for numerical evaluation has recently been presented and a computer code has been prepared for performing the calculations.[32] In the following development the solutions of Thurston are modified such that the electromechanical coupling factor in the stressed region is different from that in the unstressed region. The excess current i_c, which is an addition to the current obtained in the uncoupled approximation, is

$$i_c=(b_1+b_2)\int_0^t i(t)\,dt, \tag{6}$$

where $i(t)$ is the total current, $b_1=k_0^2B_1/t_0$, $b_2=\tilde{k}^2B_2/t_0$, k_0^2 is the electromechanical coupling factor computed to represent the effects in the unstrained region, $\tilde{k}^2$ is the electromechanical coupling factor computed to represent the effects in the strained region, and B_1 and B_2 are the acoustic impedance ratios $(1+Z_1/Z)^{-1}$ and $(1+Z_2/Z)^{-1}$ for the unstrained and strained regions, respectively, where Z is the acoustic impedance of the sample and Z_1 and Z_2 are acoustic impedances of the material in contact with the input and back electrodes, respectively.

It can be observed from Eq. (6) that the excess current due to electromechanical coupling is initially zero then changes smoothly in time. For a step-function current pulse the extra current increases linearly in time. The magnitude of the increase is directly dependent on the impedance of the materials in contact with the electrodes of the piezoelectric sample and the electromechanical coupling coefficient of the piezoelectric sample.

The theory presented in this section is applied to the determination of second- and third-order piezoelectric constants by computing the piezoelectric current from Eq. (4) for a series of experiments at different strains. The piezoelectric polarization is identified as the strain-induced contributions to displacement in Eq. (1b), and the polarization versus strain data are fit by the polynomial relation $P_i^\eta=e_{ij}\eta_j+\frac{1}{2}e_{ijk}\eta_j\eta_k$, where the abbreviated notation is applied.

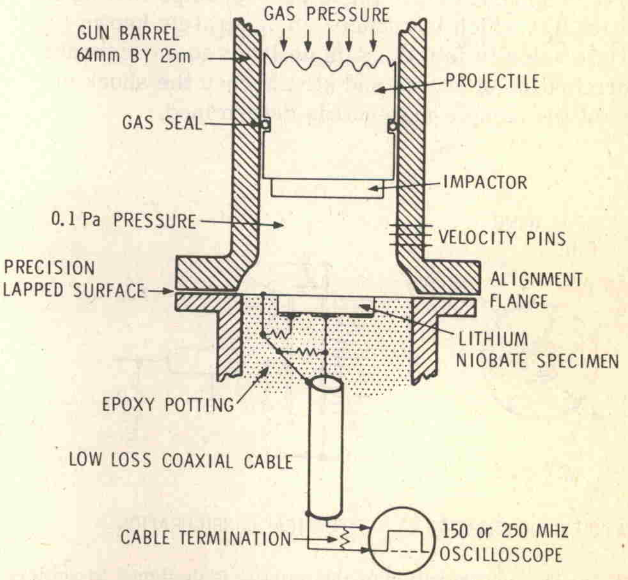


FIG. 2. The impact-loading experiment is illustrated schematically in this figure. Loading of the sample is accomplished by the precisely aligned impact of the impactor upon the sample. The current pulse from the sample is measured in a low impedance resistive circuit as the shock wave propagates through the sample.

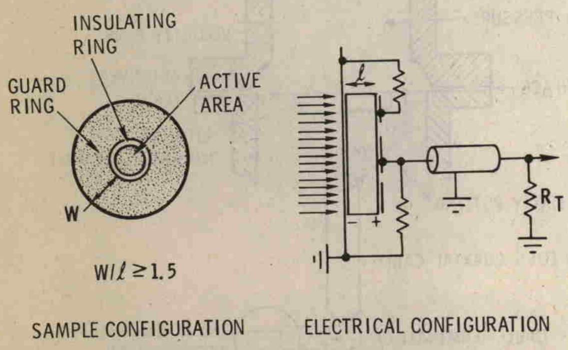
IV. EXPERIMENTAL ARRANGEMENT

The impact-loading technique for determining piezoelectric stress constants has been described in previous papers,[3,14,33] and, except for differences in detail, the present investigation follows the prior procedures.

A schematic of the impact-loading arrangement and the instrumentation is shown in Fig. 2. An impacting material whose mechanical properties have been accurately characterized was attached to the impact face of a projectile which was accelerated to a preselected velocity in a smooth-bore compressed gas gun and impacted upon the sample. Immediately prior to impact the velocity of the impactor was measured to an accuracy of ± 0.1%. The alignment between the impacting surfaces was precisely controlled such that the median value of the "tilt" or angular misalignment for the present measurements was 300 μrad.

In the present experiments most of the measurements were taken with impact velocities of from 20 to 100 m/s. In order to routinely achieve preselected velocities in this unusually low-velocity range, the gas gun[34] was modified so that the projectile could be muzzle loaded and the initial location of the projectile set at distances of from 1 to 3 m from the specimen depending upon the desired impact velocity.

The samples were loaded over large planar areas, but observations of electrical responses were limited to a central region of the sample to ensure that all measurements were taken under uniaxial strain conditions. Under these uniaxial strain conditions the conservation of mass and momentum relations lead to the expressions $S=u/U$ and $T=\rho_0 Uu$, respectively, where S is the strain and T is the component of stress in the propagation direction. (The transformation between η and S, and t and T for uniaxial strain is given in Ref. 14.) It is apparent from these relationships that an impact loading which introduces an accurately known particle velocity into the sample leads to an accurate determination of stress and strain when the shock velocity of the sample material is determined.



SAMPLE CONFIGURATION ELECTRICAL CONFIGURATION

FIG. 3. The configuration of the sample is designed to achieve one-dimensional strain and electric field conditions in the active area of the sample. The electrical configuration is a simple resistive circuit with a terminated low-loss coaxial cable transmission system. R_T is the termination resistor. The sample polarity indicated is that determined with the sample in compression.

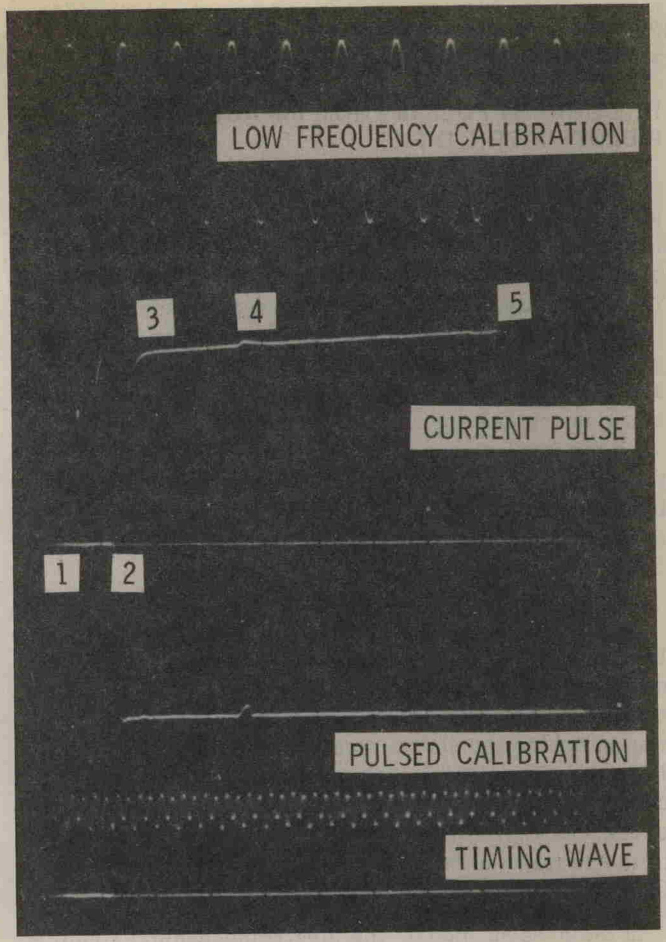


FIG. 4. Typical current pulse obtained on Z-cut lithium niobate. The current pulse is in the center of the record. Time increases from left to right. At 1 the oscilloscope is triggered. Impact begins at 2 and is complete at 3. The small perturbation at 4 is a reflection from the input termination which travels down the signal cable and back to the input. The shock wave arrives at the rear electrode at 5. The current pulse duration is 360 ns. The pulse amplitude is about 2 A from a sample with an area of 8×10^{-5} m^2 and a thickness of 2.6 mm. The impact stress is 1.2 GPa which corresponds to a strain of 4.9×10^{-3}.

Under planar impact conditions the particle velocity u, imparted to the sample by the impact, is

$$u=[(Z_I)/(Z_s+Z_I)]u_0, \tag{7}$$

where u_0 is the impact velocity, $Z_s=\rho_0 U$, the mechanical impedance of the sample at the particle velocity of the experiment, ρ_0 is the initial density, and U is the shock velocity of the sample material; similarly, Z_I is the mechanical impedance of the impactor. In the present experiments the impactors were X-cut quartz,[14] polymethyl methacrylate (PMMA),[35] or Z-cut lithium niobate.

As the shock wave propagates through the sample, an electric current is produced in an external low-impedance resistive circuit. As indicated by Eq. (3), this current pulse is approximately rectangular in shape with a pulse duration controlled by the time required for the shock wave to propagate through the thickness of the sample. In the various experiments this transit time is approximately 360—860 ns.